BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR SECONDARY CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL

MID TERM EXAMINATION (2024-25)

MATHEMATICS (041)

Class: XII Science
Date: 16/09/24
Admission Number: _____

Duration: 3 Hour
Max. Marks: 80
Roll number: ____

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub parts.

Section –A (Multiple Choice Questions)
Each question carries 1 mark

1 A relation R defined from $\{2,3,4,5\}$ *to* $\{3,6,7,10\}$ by xRy such that x is prime to y. Then, domain of R is:

(A) $\{2,3,5\}$ (B) $\{3,5\}$ (C) $\{2,3,4\}$ (D) $\{2,3,4,5\}$

2 Let f: R→R be defined by f(x) = $\frac{1}{x}$ for all x∈R. Then f is:

(A) One-One (B) Onto (C) bijective (D) Not defined

3 If y= $\sec^{-1}x$, then

(A) $0 \leq y \leq \pi$ (B) $0 < y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y < \pi$

(C) $\left\{-\frac{\pi}{2} < y < \frac{\pi}{2}\right\}$ (D) None of these

4 Principal value of $\sec^{-1}(-2)$

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) π (D) $\frac{2\pi}{5}$

5 The matrix A = $\begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a/an

(A)Square matrix (B) Diagonal Matrix

(C) Identity Matrix (D) None of these

6 If matrix A = $\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and A^2 = kA, then write the value of k

(A) 6 (B) 3 (C) 12 (D) 18

7 For what the value of x, is the matrix A = $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$, is a skew symmetric matrix?

(A) 4 (B) 2 (C) 3 (D) 1

8 If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix}$ =0 then the value of x is:

(A) 3 (B) 5 (C) 7 (D) 9

9 The area of the triangle with vertices (-3,0), (3,0) and (0,k) is 9 sq. units. The value of k will be:

(A) 9 (B) ± 3 (C) -9 (D) 6

10 For what value of x is the matrix $\begin{bmatrix} 2x+4 & 4 \\ x+5 & 3 \end{bmatrix}$ a singular matrix?

(A) 2 (B) 3 (C) 4 (D) 5

11 If the function f(x)= $\begin{cases} 3x-8\,, if\ x \le 5 \\ 2k, \quad if\ x > 5 \end{cases}$ $is\ continous, then\ the\ value\ of\ k\ is$

(A) $\frac{2}{7}$ (B) $\frac{7}{2}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$

12 If y^2 (2-x) = x^3, then $\left(\frac{dy}{dx}\right)_{(1,1)}$ is equal to

(A) 2 (B) -2 (C) 3 (D) - $\frac{3}{2}$

13 Derivative of e^{-x} is

(A) e^{-x} (B) $-e^{-x}$ (C) e^{x} (D) $-e^{x}$

14 If V= $\frac{4}{3}\pi r^3$, at what rate in cubic units is V increasing when r=10 and $\frac{dr}{dt} = 0.01$?

(A) π (B) 4π (C) 40π (D) $\frac{4\pi}{3}$

15 The function (x-sinx) decreases for

(A) All x (B) x< $\frac{\pi}{2}$ (C) 0< $x < \frac{\pi}{4}$ (D) no value of x

16 If x is real, then the minimum value of x^2-8x+17 is

(A) -1 (B) 0 (C) 1 (D) 2

17 Evaluate: $\int \frac{2}{x^2} dx$

(A) $\frac{1}{x} + c$ (B) $-\frac{1}{x} + c$ (C) $\frac{2}{x} + c$ (D) $\frac{-2}{x} + c$

18 Evaluate: $\int (3x+4)^3 dx$

(A) $\frac{(3x+4)^4}{12}+c$ (B) $\frac{(3x+4)^5}{12}+c$ (C) $\frac{(3x+5)^4}{12}+c$ (D) $\frac{(3x+4)^4}{4}+c$

Assertion and Reasoning questions: In the following two questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true and R is not the correct explanation of A.
(C) A is true but R is false.
(D) $A\ is\ false\ but\ R\ is\ true.$

19 Assertion (A): $\text{cosec}^{-1}\left(cosec\frac{9}{5}\right)=\pi-\frac{9}{5}$.

Reason (R): $\text{cosec}^{-1}(cosecx)=\pi-x, \forall\ x\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right]-\{\pi\}$

20 Assertion (A): f(x) =$|x|sinx\ is\ not\ differentiable\ at\ x=0$.

Reason (R): If f(x) is not differentiable and g(x) is differentiable at x=a, then f(x) g(x) can still be differentiable at x=a.

Section –B
[This section comprises of very short answer type questions (VSA) of 2 marks each]

21 Write the domain of the R defined on the set Z of integers as follows: (a,b)∈R such that $a^2+b^2=25$.

22 Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

OR

Find the value of $\cos^{-1}(\frac{1}{2})+2\sin^{-1}(\frac{1}{2})$.

23 Find $\frac{dy}{dx}$, if x=a $(\theta+sin\theta)$, y= a (1-cosθ)

24 The radius of a circle is increasing at the rate of 0.5cm/sec. Find the rate of increasing of its area when r is 7cm.

OR

The total cost C(x) associated with the production of x units of an item is given by $C(x)=0.005x^3-0.02x^2+30x+5000$, Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of the total cost at any level of output.

25 find $\int x\sqrt{1+2x^2}\,dx$.

Section – C
[This section comprises of short answer type questions (SA) of 3 marks each]

26 Let f: R→R be defined by f(x) = x+1. Determine whether f is onto or not.

27 Given that A= $\begin{bmatrix}2&4&0\\3&9&6\end{bmatrix}$ and B= $\begin{bmatrix}1&4\\2&8\\1&3\end{bmatrix}$ Is (AB)' = B'A'.

OR

Express $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

28 Find the matrix X for which $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$X = $\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

29 Write the value of b for which f(x) = $\begin{cases} 5x-4 & ,0<x\leq 1 \\ 4x^2+3\text{bx} & ,1<\text{x}<2 \end{cases}$ is continuous at x=1.

OR

The function f(x) is defined by f(x) = $\begin{cases} x^2+ax+b & ,0\leq x<2 \\ 3x+2 & ,2\leq x\leq 4 \\ 2ax+5b & ,4<x\leq 8 \end{cases}$ if f is continuous on[0,8],

find the values of a and b.

30 Prove that the function f(x) = tanx-4x is strictly decreasing on$\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$.

31 Evaluate: $\int \frac{x+2}{2x^2+6x+5}\,dx$

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32 Let Z be the set of integers. Show that relation R=$\{(a,b): a,b\in Z \text{ and } a+b \text{ is even}\}$ is an equivalence relation on Z.

33 Using matrix method, solve the following system of equations: x+2y+z=7, x+3z=11, 2x-3y=1.

34 Differentiate $(\sin x)^{\cos x}$ with respect to x.

OR

Differentiate $(x+1)^2(x+2)^3(x+3)^4$ with respect to x.

35 Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function f(x) = $-\frac{3}{4}x^4-8x^3-\frac{45}{2}x^2+105$.

OR

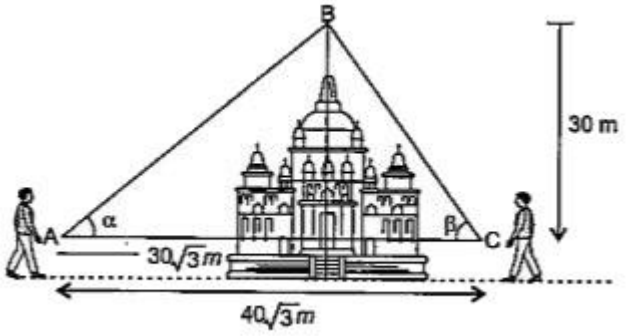
Divide 64 into two parts such that the sum of the cubes of two parts is minimum.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The all three case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36 Two men on either side of a temple, which is 30 metres high above the stairs, observe its top at the angles of elevation $\alpha \text{ and } \beta$ respectively (as shown in the figure below). The distance

between the two men is $40\sqrt{3}$ m and the distance between the first person A and the temple is $30\sqrt{3}$m.



Based on the above information. Answer the following questions.

(A) $Find\ the\ \angle CAB = \alpha$
(B) $Find\ the\ \angle ACB = \beta$
(C) $Find\ the\ \angle ABC$

37 Three shopkeepers Salim, Vijay and Venkat are using polythene bags, handmade bags (prepared by prisoners) and newspaper's envelope as carry bags. It is found that the shopkeepers Salim, Vijay and Venket are using (20, 30, 40), (30, 40, 20), (40, 20, 30) polythene bags, handmade bags and newspaper's envelops respectively. The shopkeepers Salim, Vijay and Venket spent Rs. 250, Rs. 270 and Rs. 200 on these carry bags respectively.

Using the concept of matrices and determinants, answer the following questions:

(A) What is the cost one polythene bag?
(B) What is the cost of one handmade bag?
(C) What is the cost one newspaper envelope?

38 An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 32000 litre of water that comes out from tube well.

Based on above information, answer the following questions;

(A) If the lengths, width and height of the open tank be x, x and y meter respectively, then find the total surface area of tank.

(B) Find the relation between x and y.

(C) Find the value of depth when the outer surface area of the tank will minimum.

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